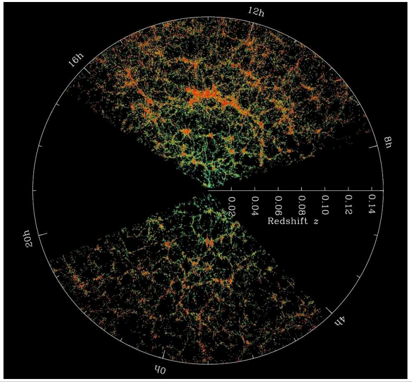

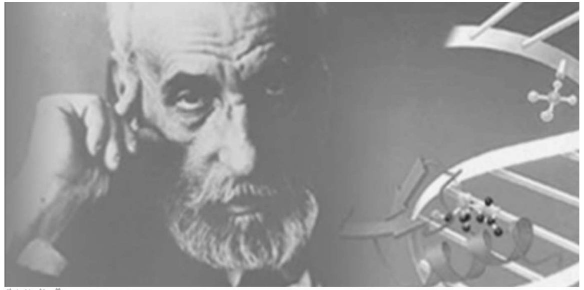

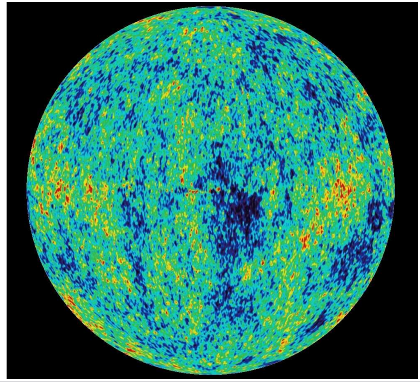

Nonsingular Universes a là Palatini.

Gonzalo J. Olmo

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> *Can we construct a Lagrangian-based phenomenological theory of gravity free from singularities and as successful as GR at low energies without introducing extra fields or new degrees of freedom?*

- A (Hamiltonian-based) example sharing this philosophy is provided by some toy models of canonical quantum gravity:
  - The effective dynamics of Loop Quantum Cosmology replaces the Big Bang singularity by a cosmic bounce using second-order equations (like GR). The bounce is due to non-perturbative quantum effects.
  - Lagrangians yielding similar dynamics could answer our question and establish a link with LQC and related approaches.

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- Since exact isotropy is a very strong idealization, here we consider the behavior of $f(R)$ and other Palatini theories in anisotropic scenarios.
- We will see that:
  - $f(R)$ models with isotropic bouncing solutions generically develop shear singularities in anisotropic scenarios.
  - Completely regular isotropic and anisotropic bouncing solutions exist in $f(R,Q)$ models, where $Q \equiv R_{(\mu\nu)}R^{(\mu\nu)}$, thus providing a promising arena to build a non-singular theory of gravity.

LQC and other bouncing $f(R)$ models

Palatini $f(R)$ theories

- Action and field equations of Palatini $f(R)$ theories:

$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu},\psi)$, where $(g_{\mu\nu},\Gamma^{\alpha}_{\beta\gamma})$ are independent.

$f_R R_{\mu\nu}(\Gamma) - \frac{1}{2}g_{\mu\nu}f(R) = \kappa^2 T_{\mu\nu}$, where $f_R \equiv df/dR$.

$\nabla_\alpha\left(\sqrt{-g}f_R g^{\beta\gamma}\right) = 0 \quad \Rightarrow \quad \Gamma^{\alpha}_{\beta\gamma} = \frac{t^{\alpha\rho}}{2}\left[\partial_\beta t_{\rho\gamma} + \partial_\gamma t_{\rho\beta} - \partial_\rho t_{\beta\gamma}\right]$, where $t_{\mu\nu} = f_R g_{\mu\nu}$.

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- Palatini $f(R)$ looks like GR with a modified source !!!

Finding the LQC effective action

- In a FRW Universe $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$. In GR $3(\dot{a}/a)^2 = \kappa^2\rho$
- For a massless scalar, the Hubble function $H = \dot{a}/a$ is given by
  - In LQC: $3H^2 = 8\pi G\rho\left(1 - \frac{\rho}{\rho_{crit}}\right)$, with $\rho_{crit} = 0.41\rho_{Planck}$.
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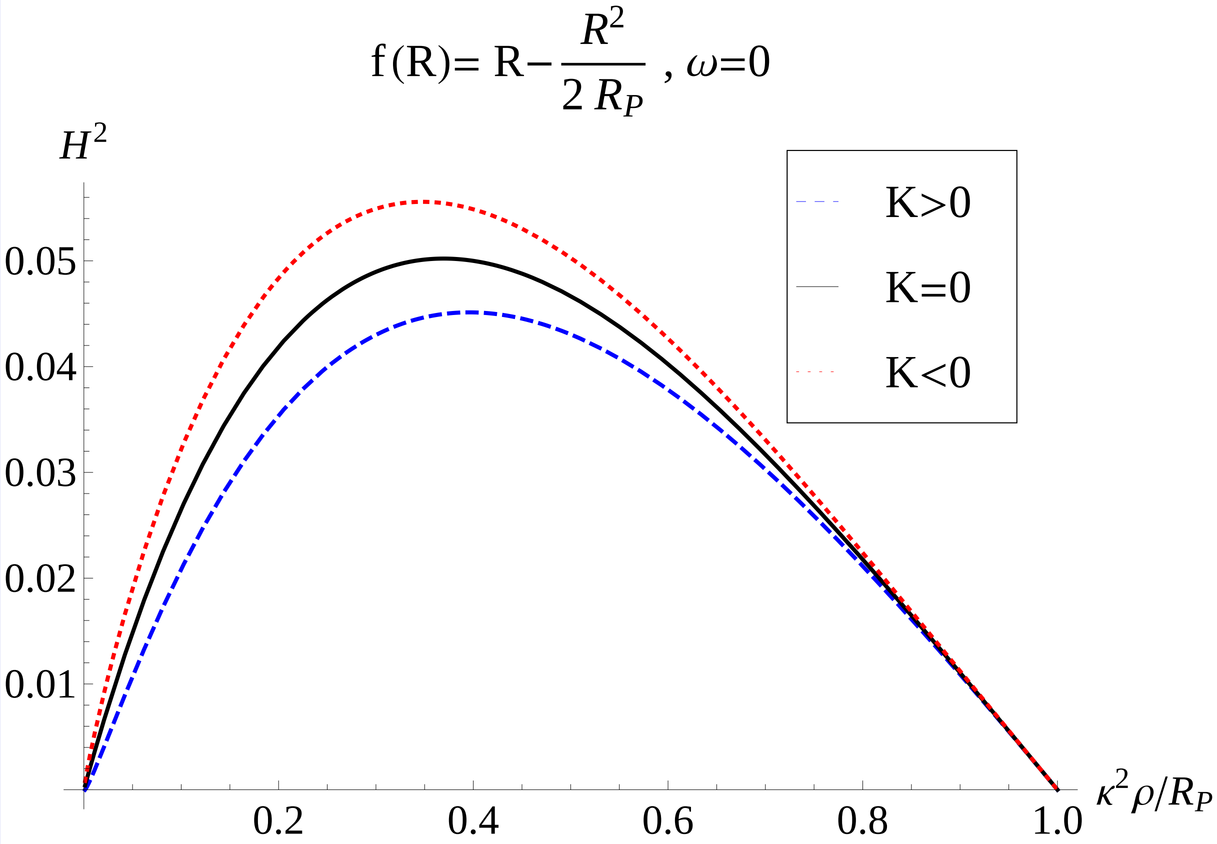
  $B=2\sqrt{\mathcal{R}_cf_R(2\mathcal{R}_cf_R-3f)}$, and $\mathcal{R}_c\equiv\kappa^2\rho_c$.
- This leads to a unique solution with $f_R\to 1$ when $R\to 0$ satisfying $\ddot{a}_{LQC}=\ddot{a}_{Pal}$ at $\rho=\rho_c$ – Olmo & Sing (2009).

Other nonsingular $f(R)$ models

- The LQC Lagrangian is not the only $f(R)$ model that avoids the Big Bang singularity. The simple model $f(R) = R + a\frac{R^2}{R_P}$ can also do the job:



- The Hubble function begins growing linearly, then reaches a maximum and drops to zero at high energies producing a cosmic bounce.

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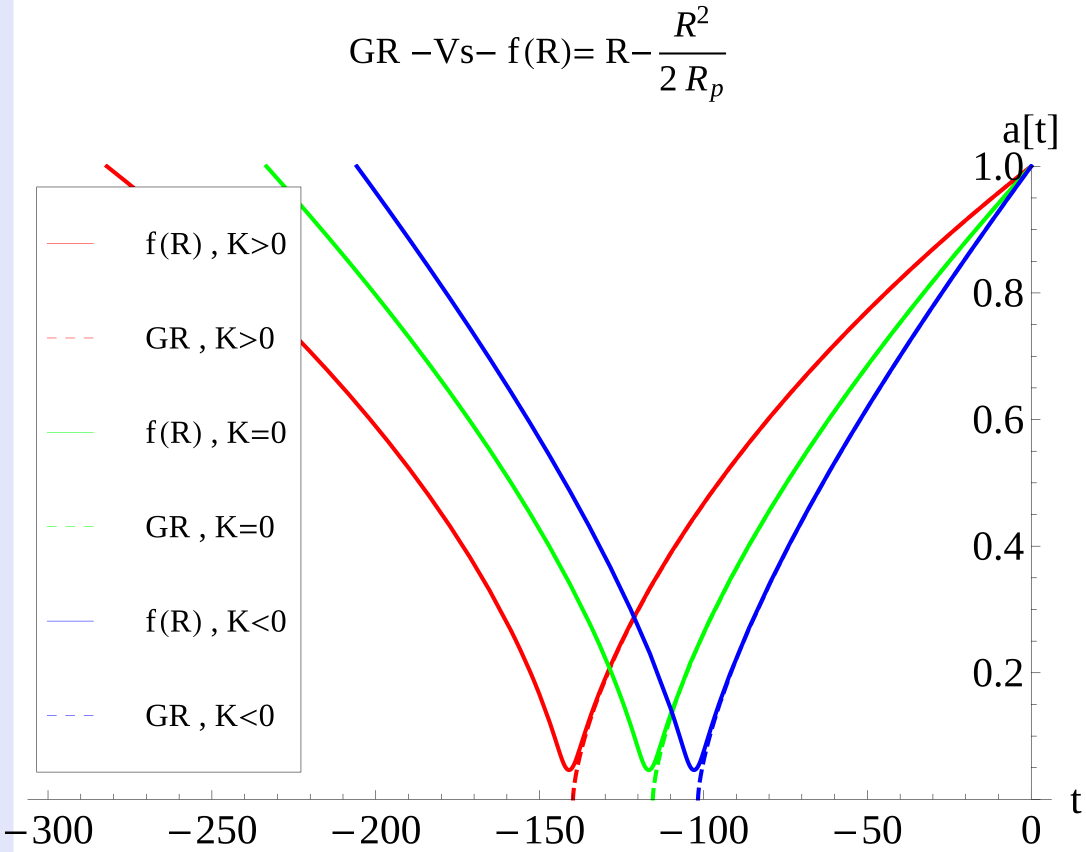

- Starting with a contracting phase, the expansion factors reach a minimum and bounce to our expanding universe.

Characterizing the $f(R)$ Bounce.

- For a general $f(R)$ theory, the Hubble function is given by ($P = w\rho$)

$$H^2 = \frac{1}{6 f_R} \frac{\left[f + \kappa^2(\rho + 3P) - \frac{6K f_R}{a^2}\right]}{\left[1 + \frac{3}{2}\tilde{\Delta}_1\right]^2}$$

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  - II: $f+\kappa^2(\rho+3P)-6Kf_R/a^2=0$.
- If $f(R)\approx R$ at low energies, only $f_R=0$ occurs. – Barragán & Olmo (2010)
  - Assuming $g(R)=2\left(1+\frac{3}{2}\tilde{\Delta}_1\right)=\frac{\left(f_{RR}[6(1+w)f-(1+3w)Rf_R]-f_R^2\right)}{f_R(Rf_{RR}-f_R)}$ such that $g(R)\approx 1$ at low R but diverges at R_P, and denoting $f=R_0e^{\lambda(R)}$, we find

$$\frac{\lambda_{RR}+\lambda_R^2}{\lambda_R^2} = \frac{[2-g(R)]}{6(1+w)-[1+3w+g(R)]R\lambda_R}$$

  - Since $R\lambda_R>0$, the denominator may vanish as $g(R)$ grows. A true bounce can only happen when $g(R)\to\infty$, but that requires that $g(R)\lambda_R$ be finite to exactly cancel out with the other terms. Since this can only happen if $\lambda_R=0=f_R$, the condition $Rf_{RR}-f_R$ is excluded.

Beyond isotropy in $f(R)$ models

Anisotropies in $f(R)$: the end of a dream.

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- Note that $R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} \sim 1/f_R^4$ confirms that the divergence of σ^2 is a true geometrical singularity.

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- Our studies have led us to the limits of $f(R)$: they are not the final answer.
- A natural next step is to explore Palatini theories with other curvature invariants. To avoid the appearance of new degrees of freedom, we restrict ourselves to theories of the form $f(R,Q)$, with $Q \equiv R_{(\mu\nu)}R^{(\mu\nu)}$

Olmo, Sanchis-Alepuz, & Tripathi (2009) ; Vitagliano, Sotiriou, & Liberati (2010) .

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- We also find that $R = R(\rho,P)$, $Q = Q(\rho,P)$, which implies a phenomenology much richer than that of $f(R)$ theories.

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- ρ and P are bounded from above: $1 - \frac{4\kappa^2(\rho+P)}{R_P} \geq 0$

We expect important changes in the dynamics at high curvatures (Big Bang, Black Holes,...).

Details of $f(R,Q)$ in FRW and Bianchi I

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- For the model $f(R,Q) = R + a\frac{R^2}{R_P} + \frac{R_{(\mu\nu)}R^{(\mu\nu)}}{R_P}$
  - If $(\Lambda_1-\Lambda_2) \to 0$ at some ρ_B then isotropic bounce is possible.
  - If $Q = Q_{max}$ a regular isotropic and anisotropic bounce is possible.

Dependence of the bounce on (a, w) in $f(R,Q)$

- The classification of the bouncing solutions for $f(R,Q) = R + a\frac{R^2}{R_P} + \frac{R_{\mu\nu}R^{\mu\nu}}{R_P}$ is as follows:

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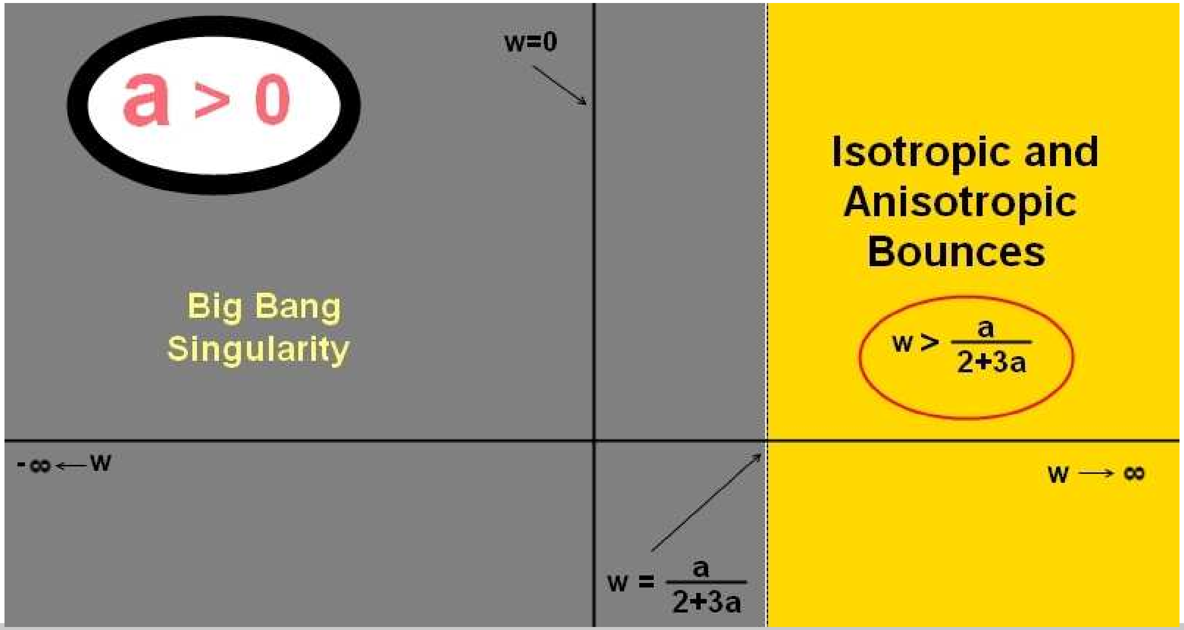
- If $a > 0$ the bounce occurs when $Q = Q_{max}$. This is so because $\partial_\rho \Omega \sim \partial_\rho \lambda \sim \partial_\rho Q$ and Q contains a term of the form $\sqrt{\Phi}$ which vanishes at Q_{max}. The density at the maximum is given by

$$\frac{\kappa^2 \rho_{Q_{max}}}{R_P} \equiv \frac{1+5w-2a(1-3w)-\sqrt{8(1+w)(2w-a(1-3w))}}{(1+2a)^2(1-3w)^2}$$

  - The bounce occurs at that density if $w > \frac{a}{2+3a}$



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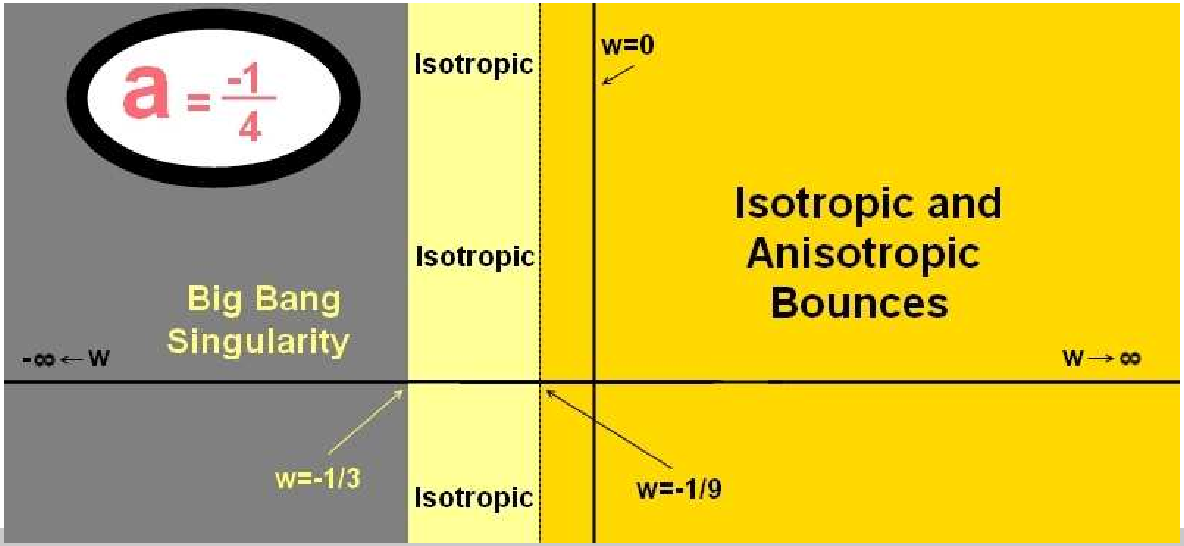
- The classification of the bouncing solutions for $f(R,Q) = R + a\frac{R^2}{R_P} + \frac{R_{\mu\nu}R^{\mu\nu}}{R_P}$ is as follows:
- If $a \leq 0$ the bounce occurs at the following density:

$$\frac{\kappa^2\rho_B}{R_P} = \begin{cases} \frac{1+6w-2a(1-3w)-3\sqrt{w(2+3w)-a(1+w)(1-3w)}}{(1+a)(1+4a)(1-3w)^2} & \text{if } w \leq w_0 \\ \frac{\kappa^2\rho_{Q_{max}}}{R_P} & \text{if } w \geq w_0 \end{cases}$$

  - Note that w_0 is always negative.
  - For $w \geq w_0$ the bounce is due to reaching Q_{max}.
  - For $w \leq w_0$ the bounce is due to the vanishing of $\Lambda_1 - \Lambda_2$.



Details of $a \leq 0$ isotropic bounces

- How negative can w be extended beyond the matching point w_0?
- If $-1/4 < a \leq 0$ restricted by the argument of the square root for $w \leq w_0 \Rightarrow -\frac{1}{3} + \frac{1}{3}\sqrt{\frac{1+4a}{1+a}} < w < \infty$
- If $a = -1/4$ the density at the bounce is given by

$$\frac{\kappa^2 \rho_B}{R_P} = \begin{cases} \frac{1}{3(1+3w)} & \text{if } w \leq -\frac{1}{9} \\ \frac{\kappa^2 \rho_{Qmax}}{R_P} & \text{if } w \geq -\frac{1}{9} \end{cases} \Rightarrow -1/3 \leq w < \infty$$

- If $-1/3 \leq a \leq -1/4$ Though here the square root is always real, we find numerically that the bouncing solutions cannot be extended beyond the value $w < -1$, where ρ_B reaches a maximum $\Rightarrow -1 < w < \infty$
- If $-1 \leq a \leq -1/3$ here $-1 < w$ also. We also find restrictions for $w > 1$ due to zeros in the denominator of H^2. $\Rightarrow -1 < w < \frac{\alpha+\beta a}{(1+3a)^2} > 1$, where $\alpha = 1.1335$ and $\beta = -3.3608$.
- If $a \leq -1$ w depends on the square root $\Rightarrow -1 < w < a/(2+3a)$.

Note that dust and radiation are always non-singular!!!

Conclusions and Perspectives

- Palatini theories have an extraordinary ability to avoid singularities without the need for extra degrees of freedom:
  - Using $f(R)$ Lagrangians we reproduced the isotropic LQC dynamics.
  - Other simple models, $f(R) = R + R^2/R_P$, also avoid the big bang.

$f(R,Q)$ Lagrangians generate bouncing solutions in anisotropic scenarios and for standard sources of matter and radiation, $0 \leq w \leq 1/3$ *!!!*

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- The independent connection is fundamental to avoid new degrees of freedom and yield non-linear matter contributions that generate the bounce.
- Natural future directions:
  - Cosmology of other Bianchi models.
  - Gravitational collapse and structure of compact objects in $f(R,Q)$.
  - Exploration of more general quadratic Lagrangians.
  - Hamiltonian description of general Palatini theories.

● Motivation and Summary

Thanks !!!